Momentum and Energy Transfer in Turbulent Pipe Flow: The Penetration Model Revisited

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The extreme difficulties involved in trying to describe mathematically heat, mass, and momentum transfer in turbulent pipe flow are well-known, and for practical calculations one is forced to fall back on the empirical correlations. Our qualitative understanding of the turbulent exchange process has increased significantly in recent years, however, due to the careful studies of Corino and Brodkey (1969), Kline et al. (1967), Popovich and Hummel (1967), and many others. These researchers have clearly described the unsteady nature of the viscous sublayer and identified the bursting process as a key event in turbulent flow.

Corino and Brodkey (1969) offer a lucid picture of the wall region in turbulent pipe flow reminiscent in many ways of the early penetration model of Einstein and Li (1956). In this model, a lump of fluid with axial velocity u_0 suddenly penetrates into the region immediately adjacent to the pipe wall. The velocity profile then decays according to

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \qquad (1)$$

$$y = 0 \qquad u = 0$$

$$y \to \infty \qquad u \to u_0$$

$$t = 0 \qquad u = u_0$$

for a time period θ, following which the process begins anew. It is important to emphasize that the picture suggested by Equation (1) represents an extreme idealization of the process described by Corino and Brodkey. In spite of this, the model accurately describes a large variety of experimental data.

The quantity u_0 in Equation (1) was originally chosen by Einstein and Li (1956) as the velocity at a distance equal to three times the displacement thickness from the wall. More recently, Meek and Baer (1970) have evaluated u_0 by coupling the velocity profile in the sublayer to the velocity profile in the core obtained via a mixing length formulation. Integration over the cross section of the pipe then yields a relation between u_0 and the average fluid velocity u_b. This approach does eliminate the arbitrariness associated with the definition of u_0 but complicates the mathematical details of the theory and distracts from its conceptual simplicity. The studies of Corino and Brodkey suggest that u_0 is of order u_b, as would be anticipated on purely physical grounds. It is therefore natural, and consistent with the crude nature of the model, to simply equate these two velocities. This simplification greatly reduces the algebraic and numerical difficulties associated with the model and leads to predictions equivalent to and in some instances superior to the Meek and Baer (1970) formulation. Furthermore, the model as thus formulated is useful in interpreting various aspects of turbulent drag reduction.

MOMENTUM TRANSFER

Equation (1), with the condition $u_0 = u_b$, yields the following alternative expressions for the friction factor:

$$f = \frac{4}{u_b}\sqrt{\frac{\nu}{\pi\theta}} \qquad (2)$$

$$f = \frac{8}{\pi\theta_+^2} \qquad (3)$$

where

$$\theta_+ = \left(\frac{\theta}{\nu}\right)^{1/2} u_*$$

is a dimensionless contact time. Equation (2) indicates that at constant bulk velocity, the energy dissipation in turbulent pipe flow is an inverse function of the square root of the contact time, or time between turbulent bursts. Current theories of drag reduction in dilute polymer solutions suggest that the primary function of the macromolecule is to reduce the bursting frequency and hence increase θ_+. This is a consequence of the fact that the bursting process consists of a rapid outflow between pairs of longitudinally oriented, counter-rotating eddies (Fortuna and Hanratty, 1972). Such a flow field is known as an extensional flow (Seyer and Metzner, 1969), and dilute polymer solutions are known to exhibit unusually large resistances to such flows (Seyer and Metzner, 1969; Metzner and Metzner, 1970; Everage and Gordon, 1971). On the other hand, for shearing flows such as the flow field described by Equation (1), the polymer concentrations used for drag reduction are normally too small to have any appreciable effect. Consequently, Equation (2) and (3) should apply to both Newtonian and drag reducing systems, the only difference being the reduced values of f and increased values of θ for the polymer solutions. Figure 1 illustrates this point. Here we have plotted θ_+ versus N_{Re} for Newtonian solutions and for Virk's maximum drag reduction asymptote (Virk et al., 1967)

$$f = 0.42\ N_{Re}^{-0.55}$$

The values of θ_+ obtained for the Newtonian fluid vary from 19 at $N_{Re} = 10^4$ to 24 at $N_{Re} = 10^5$, while in the Meek and Baer formulation, θ_+ is constant at a value of 18 for Reynolds numbers greater than 10^4. Meek (1972) has recently compiled an extensive list of available experimental values of the dimensionless sublayer period versus N_{Re}. The great bulk of the data is for Reynolds numbers of $10^4 \to 10^5$, with values of θ_+ scattered randomly between approximately 15 to 24. Hence, it is not possible

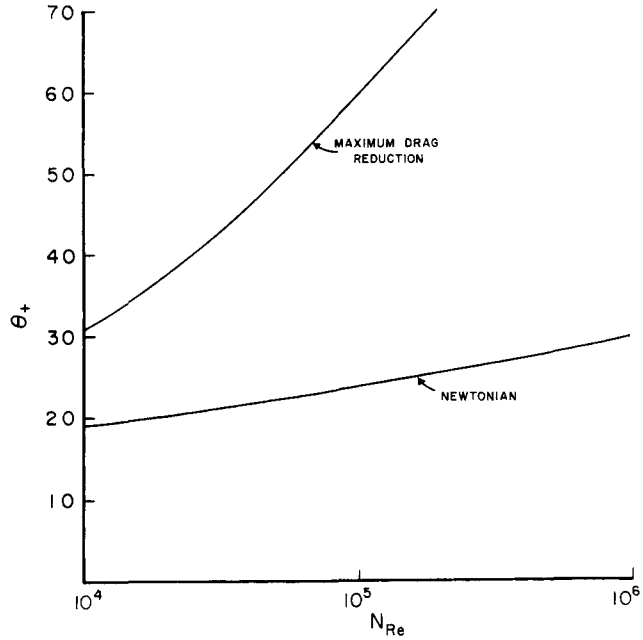

Fig. 1. Dimensionless contact time between turbulent bursts vs. Reynolds number for Newtonian fluids and for maximum drag reduction.

to express a preference for either the Meek and Baer formulation or the present formulation based on the experimental bursting period (sublayer period) data. The encouraging factor is that both formulations give values for θ_+ which lie within the range of the experimental measurements.

Equation (2) allows one to calculate the percent drag reduction at constant bulk velocity if the increase in θ following the addition of the polymer is known (It is assumed that $\nu = \nu_p$):

$$\% \text{ DR} = 100\left(1 - \frac{f_p}{f}\bigg|_{\text{const. } u_b}\right) = 100\left(1 - \sqrt{\frac{\theta}{\theta_p}}\right) \tag{4}$$

At present, there are no available data for turbulent pipe flow with which this expression could be checked. However, Fortuna and Hanratty (1971) have reported values of percent drag reduction and change in dimensionless scale size in the transverse direction for aqueous solutions of polyacrylamide. If we assume that the scale size in the axial direction increases by the same proportion as that in the transverse direction, then the increase in the area/burst can be calculated. This in turn should be roughly proportional to the increase in bursting time so that an estimate of the change in θ may be calculated, and from Equation (4) the percent drag reduction. The results of this procedure are listed in Table 1, where it is clear that the agreement between the penetration theory and the experimental data is quite good.

HEAT TRANSFER

The heat transfer equation analogous to Equation (1) is

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{5}$$

$$y = 0 \qquad T = T_w$$

$$y \to \infty \qquad T \to T_0$$

$$t = 0 \qquad T = T_0$$

Table 1. Comparison Between Predicted Results of Penetration Model and Data of Fortuna and Hanratty (1972)

Relative scale size in transverse direction (Fortuna and Hanratty)	% DR (Fortuna and Hanratty)	% DR (Penetration theory, $u_0 = u_b$)
1.0	0	0
2.1	27.0	31.0
2.9	34.6	41.3
11	64.5	69.8

where T_w is the wall temperature, and T_0 (analogous to u_0) is taken here as the bulk fluid temperature T_b. The solution to Equation (4) in conjunction with Figure 1 yields expressions for the Stanton number which agree closely with Deissler's (1955) well-known correlations, but only for small values of N_{Pr} (less than 2.0). Meek and Baer (1970) experienced similar problems with their formulation* and suggested that the introduction of a nonzero penetration thickness was necessary. Choosing a value of $\delta_+ = 1.5$ brought their model predictions into fairly close agreement with the Deissler correlation up to Prandtl numbers of about 100.

In the present formulation, it is possible to obtain an analytical expression for N_{St} for penetration to a distance δ from the tube wall:

$$N_{St} = \frac{2N_{Pr}^{-1}}{u_{b+}\,\delta_+\,\theta_+^2}\int_0^{\theta_+} \text{erf}\left(\frac{\delta_+ N_{Pr}^{1/2}}{2s}\right) s\, ds$$

$$\delta_+ = \frac{\delta u_*}{\nu}$$

Choosing a value of $\delta_+ = 1.5$ leads to numerical results quite similar to those of Meek and Baer (1970). A slight deviation of the results from the Deissler curves, which was most pronounced (but never large) in the range $10 < N_{Pr} < 10^2$, suggested to us a dependence of δ_+ on N_{Pr}. The expression

$$\delta_+ = \frac{4.0}{N_{Pr}^{1/7}}$$

yielded predictions in excellent agreement with the Deissler correlation as illustrated in Figure 2. The effect of a nonzero δ_+ on the friction factor $-\theta_+$ relationship was found to be negligible.

Figure 3 is a plot of $f/2$ and $N_{St}\,N_{Pr}^{0.667}$ versus N_{Re} for Newtonian solutions, and for solutions exhibiting maximum drag reduction. For the Newtonian solutions, the Chilton-Colburn analogy ($N_{St}\,N_{Pr}^{0.667} = f/2$) is predicted within $\pm 50\%$ (usually the deviation is much smaller than this) over a range of Prandtl Numbers from 1 to 5000, and Reynolds Numbers from 10^4 to 5×10^5. Hubbard and Lightfoot (1966) found the Chilton-Colburn analogy to correlate heat and mass transfer data over a range of Prandtl Numbers from 10 to 5000, and Reynolds Numbers from 10^4 to 5×10^4. On the other hand, the j-factor relationship is not predicted to hold for the drag-reducing solutions, except for small ($\cong 1.0$) value of N_{Pr}, in disagreement with available data (Smith et al., 1969). This difficulty is perhaps due to the increased δ_+ observed in such systems (Arunachalam et al., 1972).

* Note that Meek and Baer obtained T_0 by coupling a mixing length formulation in the core to the sublayer equations.

CONCLUSIONS

1. The penetration model of heat and momentum (and mass) transfer in turbulent pipe flow, although a very coarse approximation of the true physical system, accurately describes many of the macroscopic aspects of these processes for both Newtonian and drag reducing fluids.

2. The simplest form of the penetration model, that where the initial penetration possesses the bulk or mean fluid properties, gives results closely equivalent and in some cases superior to those of the more complex Meek and Baer formulation, as well as yielding a closed-form solution for N_{St} versus N_{Pr} and N_{Re} in the case of a non-zero penetration length.

3. The results of this model suggest that theoretical studies of turbulent drag reduction should be directed toward the influence of high molecular weight linear polymers on transient stretching flows, in order to quantify the reduction in bursting frequency.

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NOTATION

f = friction factor for Newtonian fluid
f_p = friction factor for drag reducing fluid
t = time
T = temperature
u = fluid velocity
u_b = average velocity
u_{b+} = u_b/u_*
u_0 = velocity of penetration in Einstein-Li model
u_* = friction velocity
y = distance from tube wall
N_{Pr} = Prandtl number
N_{Re} = Reynolds number
N_{St} = Stanton number
δ = closest distance of approach to the wall of a fluid penetration
δ_+ = $\delta u^*/\nu$

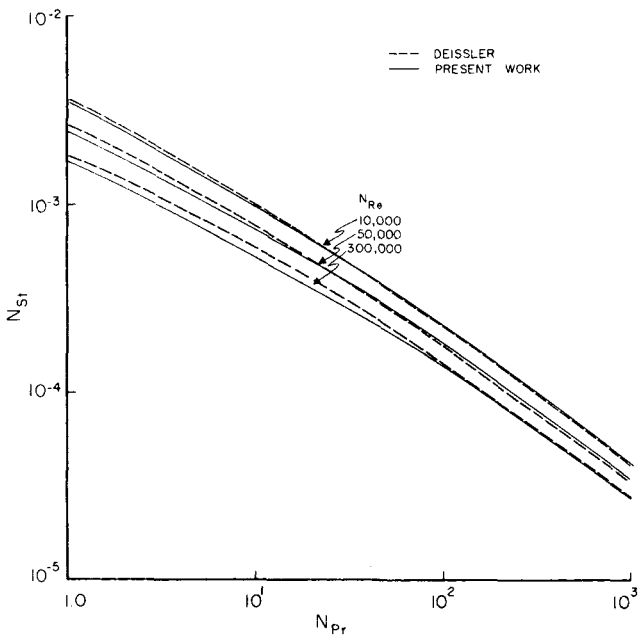

Fig. 2. Comparison of Stanton number predictions of present theory with Deissler (1955) correlation.

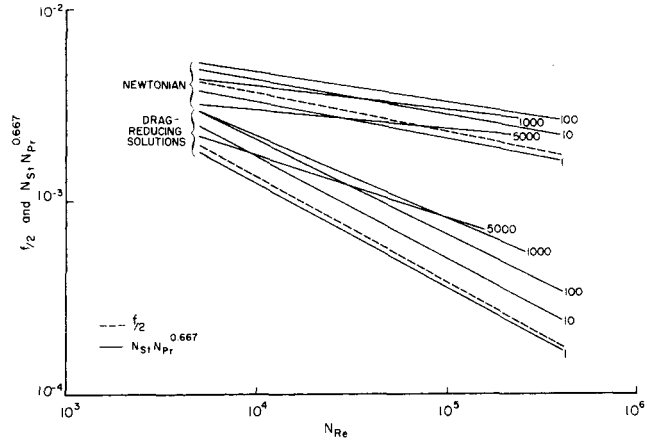

Fig. 3. $f/2$ and $N_{St}\ N_{Pr}^{0.667}$ vs. N_{Re} for Newtonian and maximum drag-reducing fluids as predicted by present theory. Numbers by curves refer to value of N_{Pr}.

θ = time between turbulent bursts for a Newtonian fluid
θ_p = time between turbulent bursts for a drag reducing solution
ν = kinematic viscosity of Newtonian solution
ν_p = kinematic viscosity of polymer solution

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